Inducing Decision Trees

In this homework you will implement and test the decision tree learning algorithm (See Mitchell, Chapter 3). It is acceptable to look at Java code for decision trees in WEKA. However, you cannot copy code from WEKA.

You can use either C/C++, Java or Python to implement your algorithms. Your C/C++ implementations should compile on Linux gcc/g++ compilers.

- Download the two datasets available on the class web page. Each data set is divided into three sets: the training set, the validation set and the test set. Data sets are in CSV format. The first line in the file gives the attribute names. Each line after that is a training (or test) example that contains a list of attribute values separated by a comma. The last attribute is the class-variable. Assume that all attributes take values from the domain \{0,1\}.

- Implement the decision tree learning algorithm. As discussed in class, the main step in decision tree learning is choosing the next attribute to split on. Implement the following two heuristics for selecting the next attribute.

  1. Information gain heuristic (See Class slides, Mitchell Chapter 3).
  2. Variance impurity heuristic described below.

  Let $K$ denote the number of examples in the training set. Let $K_0$ denote the number of training examples that have $class = 0$ and...
\[ K_1 \text{ denote the number of training examples that have } \text{class} = 1. \]
The variance impurity of the training set \( S \) is defined as:

\[
VI(S) = \frac{K_0 K_1}{K} K
\]

Notice that the impurity is 0 when the data is pure. The gain for this impurity is defined as usual.

\[
Gain(S, X) = VI(S) - \sum_{x \in \text{Values}(X)} \Pr(x)VI(S_x)
\]

where \( X \) is an attribute, \( S_x \) denotes the set of training examples that have \( X = x \) and \( \Pr(x) \) is the fraction of the training examples that have \( X = x \) (i.e., the number of training examples that have \( X = x \) divided by the number of training examples in \( S \)).

- Implement the post pruning algorithm given below as Algorithm 1 (See also Mitchell, Chapter 3).

- Implement a function to print the decision tree to standard output. We will use the following format.

```
weasley = 0 :
| | honor = 0 :
| | | barclay = 0 : 1
| | | barclay = 1 : 0
| | honor = 1 :
| | tea = 0 : 0
| | tea = 1 : 1
weasley = 1 : 0
```

According to this tree, if weasley = 0 and honor = 0 and barclay = 0, then the class value of the corresponding instance should be 1. In other words, the value appearing before a colon is an attribute value, and the value appearing after a colon is a class value.

- Once we compile your code, we should be able to run it from the command line. Your program should take as input the following six arguments:
Algorithm 1: Post Pruning

Input: An integer \( L \) and an integer \( K \)
Output: A post-pruned Decision Tree

begin

Build a decision tree using all the training data. Call it \( D \);
Let \( D_{\text{Best}} = D \);

for \( i = 1 \) to \( L \) do

Copy the tree \( D \) into a new tree \( D' \);
\( M \) = a random number between 1 and \( K \);

for \( j = 1 \) to \( M \) do

Let \( N \) denote the number of non-leaf nodes in the decision tree \( D' \).
Order the nodes in \( D' \) from 1 to \( N \);
\( P \) = a random number between 1 and \( N \);
Replace the subtree rooted at \( P \) in \( D' \) by a leaf node.
Assign the majority class of the subset of the data at \( P \) to the leaf node.;

/* For instance, if the subset of the data at \( P \) contains 10 examples with \( \text{class} = 0 \) and 15 examples with \( \text{class} = 1 \), replace \( P \) by \( \text{class} = 1 \) */

end

Evaluate the accuracy of \( D' \) on the validation set;
/* accuracy = percentage of correctly classified examples */

if \( D' \) is more accurate than \( D_{\text{Best}} \) then

\( D_{\text{Best}} = D' \);

end

end

return \( D_{\text{Best}} \);

end
\texttt{.\ program \ <L> \ <K> \ <training-set> \ <validation-set> \ <test-set> \ <to-print>}
\texttt{L: integer (used in the post-pruning algorithm)}
\texttt{K: integer (used in the post-pruning algorithm)}
\texttt{to-print:{yes,no}}

It should output the accuracies on the test set for decision trees constructed using the two heuristics as well as the accuracies for their post-pruned versions for the given values of L and K. If to-print equals yes, it should print the decision tree in the format described above to the standard output.

\textbf{What to Turn in}

\begin{itemize}
  \item Your code and a Readme file for compiling the code.
\end{itemize}

On the two datasets available on the class web page:

\begin{itemize}
  \item Report the accuracy on the test set for decision trees constructed using the two heuristics mentioned above.
  \item Choose 10 suitable values for \(L\) and \(K\) (not 10 values for each, just 10 combinations). For each of them, report the accuracies for the post-pruned decision trees constructed using the two heuristics.
\end{itemize}
Additional Questions

Answers to the following questions will not be graded. However, they will serve as practice questions for your midterm/final.

1. Represent the following functions using decision trees: \( Y = A \lor B \), \( Y = (A \lor B) \land (B \lor C) \land (A \lor C) \) and \( Y = (A \lor B) \land \neg A \land \neg B \)

2. Compute the information gain of Attribute A.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>+</td>
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<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

3. What will be the training set error for a decision tree classifier on this training data.

4. True or False? Can you always convert any arbitrary non-binary decision tree to a binary decision tree. A binary decision tree is a decision tree in which all splits are binary (i.e., branching factor = 2). Justify your answer.